CBCS SCHEME

USN | | | | | | | | | | | **15CS36**

Third Semester B.E. Degree Examination, Aug./Sept. 2020
Discrete Mathematical Structures

Time: 3 hrs.   Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Define tautology. Prove that, for any propositions p, q, r the compound proposition

$[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology. **(05 Marks)**

b. Prove the following logical equivalences using laws of logic.

i) $[p \vee q \vee (\sim p \wedge \sim q \wedge r)] \Leftrightarrow (p \vee q \vee r)$

ii) $[(\sim p \vee \sim q) \to (p \wedge q \wedge r)] \Leftrightarrow p \wedge q.$ **(05 Marks)**

c. Find whether the following argument is valid :
No engineering student of first or second semester studies logic.
<u>Anil is an engineering student who studies logic</u>
$\therefore$ Anil is not in second semester **(06 Marks)**

OR

2 a. If a proposition q has the truth value 1, determine all truth value assignments for the primitive propositions p, r and s for which the truth value of the following compound proposition is 1.

$[q \to \{(\sim p \vee r) \wedge \sim s\}]\{\wedge s \to (\sim r \wedge q)\}.$ **(04 Marks)**

b. Establish the validity of the following arguments :

$\forall x, [p(x) \vee q(x)]$
$\underline{\forall x, [\{\sim p(x) \wedge q(x)\} \to r(x)]}$
$\therefore \forall x, [\sim r(x) \to p(x)]$ **(06 Marks)**

c. Give :
i) A direct proof
ii) An indirect proof
iii) Proof by contradiction for the following statement :

"If n is an odd integer, then n + 9 is an even integer". **(06 Marks)**

Module-2

3 a. If n is any positive integer, prove that $1.2 + 2.3 + 3.4 + \ldots + n(n+1) = \frac{1}{3}n(n+1)(n+2)$, using mathematical induction. **(05 Marks)**

b. A bit is either 0 or 1. A byte is a sequence of 8 bits. Find :
(i) the number of bytes, (ii) the number of bytes that being with 11 and end with 11 (iii) the number of bytes that begin with 11 and do not end with 11 (iv) number of bytes that begin with 11 end with 11. **(06 Marks)**

c. Find the number of arrangements of all the letters in TALLAHASSEE. How many of these arrangements have no adjacent A's. **(05 Marks)**

OR

4 a. Prove by mathematical induction that, for every positive integer n, 5 divides $n^5 - n$. **(05 Marks)**

b. Find an explicit definition of the sequence defined recursively by

$a_1 = 7$, $a_n = 2a_{n-1} + 1$ for $n \geq 2$. **(05 Marks)**

c. In how many ways can one distribute eight identical balls into four distinct containers so that :

i) No container is left empty?

ii) The fourth container gets an odd number of balls. **(06 Marks)**

Module-3

5 a. Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} 3x - 5 & \text{for} \quad x > 0 \\ -3x + 1 & \text{for} \quad x \leq 0 \end{cases}$$

Determine : $f(0)$, $f(-5/3)$, $f^{-1}(0)$, $f^{-1}(3)$, $f^{-1}([-5, 5])$. **(06 Marks)**

b. Prove that if 101 integers are selected from the set S = {1, 2, 3, . . . , 200}, then at least two of these are such that one divides the other. **(04 Marks)**

c. Let A = {1, 2, 3, 4, 5}. Define a relation R on $A \times A$ by $(x_1, y_1)\ R(x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$.

i) Verify that R is an equivalence relation on $A \times A$

ii) Determine the equivalence classes [(1, 3)], [(2, 4)], [(1, 1)]

iii) Determine the partition of $A \times A$ induced by R. **(06 Marks)**

OR

6 a. Suppose that a patient is given a prescription of 45 pills with instructions to take at least one pill a day for 30 days. Prove that there must be a period of consecutive days during which the patient takes a total of exactly 14 pills. **(05 Marks)**

b. Let $f : A \to B$, $g : B \to C$ and $h : C \to D$ be three functions. Then prove that $(h \circ g) \circ f = h \circ (g \circ f)$. **(06 Marks)**

c. For A = {a, b, c, d, e}, the Hasse diagram for the poset (A, R) is as shown below : (Ref. Fig.Q6(c))

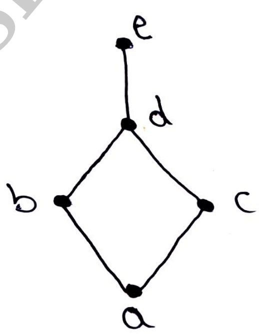

Fig.Q6(c)

Determine the relation matrix for R and construct the digraph for R. **(05 Marks)**

Module-4

7 a. Out of 30 students in a hostel, 15 study history, 8 study economics, and 6 study geography. It is known that 3 students study all these subjects. Show that 7 or more students study none of these subjects. **(05 Marks)**

b. By using the expansion formula, obtain the rook polynomial for the board C shown below :

		1
	2	3
4	5	6
7	8	

(05 Marks)

c. There are 3 pegs fixed vertically on a table, and n circular disks having holes at their centers and having increasing diameters are slipped onto one of these pegs, with the largest disk at the bottom. The disks are to be transferred, one at a time, onto another peg with the condition that at no time a larger disk is put on a smaller disk. Determine the number of moves for the transfer of all the n disks, so that at the end the disks are in their original order. **(06 Marks)**

OR

8 a. Determine the number of positive integers n such that $1 \le n \le 100$ and n is not divisible by 2, 3 or 5. **(05 Marks)**

b. Four persons P_1, P_2, P_3, P_4 who arrive late for a dinner party find the that only one chair at each of five tables T_1, T_2, T_3, T_4 and T_5 is vacant. P_1 will not sit at T_1 or T_2, P_2 will not sit at T_2, P_3 will not sit at T_3 or T_4 and P_4 will not sit at T_4 or T_5. Find the number of ways they can occupy the vacant chairs. **(06 Marks)**

c. Solve the recurrence relation $F_{n+2} = F_{nH}$ for $n \ge 0$, given $F_0 = 0$, $F_1 = 1$. **(05 Marks)**

Module-5

9 a. For a graph with n vertices and m edges, if δ is the minimum and Δ is the maximum of the degrees of vertices, show that $\delta \le \frac{2m}{n} \le \Delta$. **(05 Marks)**

b. Define isomorphism. Show that the following two graphs are isomorphic.(Ref. Fig.Q9(b)).

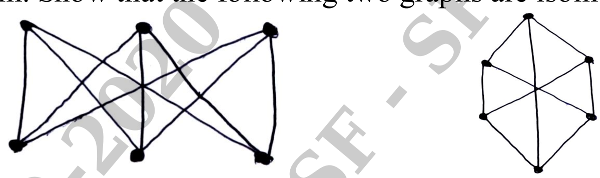

Fig.Q9(b) **(06 Marks)**

c. Prove that a tree with two or more vertices contains at least two leaves (pendant vertices). **(05 Marks)**

OR

10 a. Define each of the following with an example :
i) Regular graph
ii) Bipartite graph
iii) Complete bipartite graph. **(05 Marks)**

b. Suppose that a tree T has N_1 vertices of degree 1, N_2 vertices of degree 2, N_3 vertices of degree 3, . . ., N_K vertices of degree K. Prove that :

$$N_1 = 2 + N_3 + 2N_4 + 3N_5 + \ldots + (K-2)N_K.$$

(05 Marks)

c. Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively. **(06 Marks)**

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